$F_w(Y)$

$$\max_w \prod_i P_w(y^i) \qquad P(y^i \mid W)$$

$$\min \; -\log \prod_i P_w(y^i) = \sum_i -\frac{1}{\beta} \log P_w(y^i)$$

$$P_w(y^i) = \frac{e^{-\beta F_w(y^i)}}{\int_y e^{-\beta F_w(y^i)}}$$

$$\mathcal{L}(w) = \frac{1}{P} \sum_i^P \left[F_w(y^i) + \frac{1}{\beta} \log \int_y e^{-\beta F_w(y)} \right]$$

$$\frac{\partial \mathcal{L}}{\partial w} = \frac{1}{P} \sum_i^P \frac{\partial F(y^i)}{\partial w} - \int_y P_w(y) \frac{\partial F(y)}{\partial w}$$

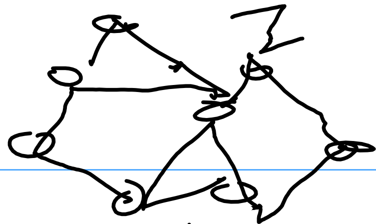
Monte Carlo Markov chain MC

MCMC

$\bar{y}^i$ MC

$$w \leftarrow w - \eta \left(\frac{\partial F(y^i)}{\partial w} - \frac{\partial F(\bar{y}^i)}{\partial w} \right)$$

- Product of Experts
- Boltzmann Machines, RBM



$$E(z) = -z^T W z$$

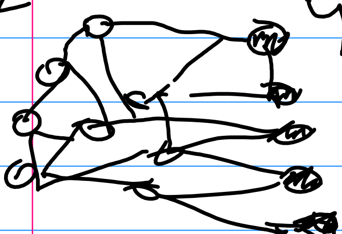
1982 Hopfield Net $\quad z_i = \sigma\left(\sum_j w_{ij} z_j\right)$

$$\mathcal{L}(w) = \frac{1}{p}\sum_k E(z^k) \qquad W \leftarrow W - \frac{\partial E}{\partial W}$$

$$w_{ij} \leftarrow w_{ij} + \eta z_i^k z_j^k$$

Herding Max Welling

1983 Hinton, Sejnowsky Boltzmann Machine



$y^k \quad z^k \qquad F(y) = \min_z E(y,z)$

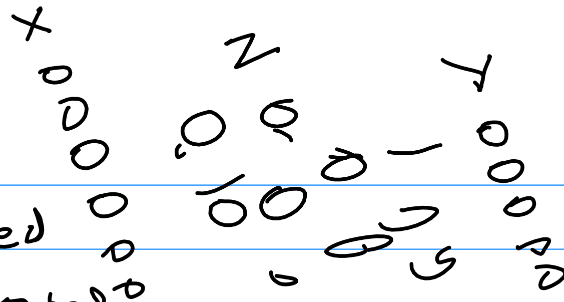
$$F(y) = -\frac{1}{\beta}\log\sum_z e^{-\beta E(y,z)}$$

$\check{y}^k \quad \check{z}^k$

$$w_{ij} = w_{ij} - \eta\left(-z_i^k z_j^k + \check{z}_i^k \check{z}_j^k\right)$$

$$w_{ij} = w_{ij} + \eta\left(z_i^k z_j^k - \check{z}_i^k \check{z}_j^k\right)$$

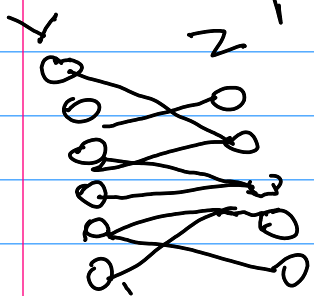
Supervised BM



X: inputs, always clamped

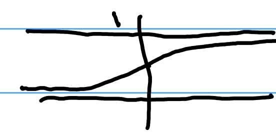
Z: hidden, never clamped

Y: outputs clamped/unclamped



Restricted Boltzmann machine

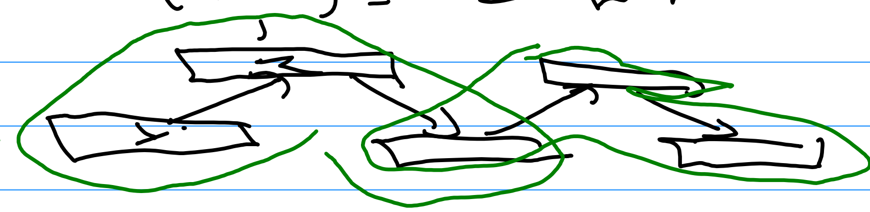
$$\sigma(a) = \frac{1}{1+e^{-\beta a}}$$



if $\sigma\left(\sum_j w_{ij} y_j\right) < \mathrm{Rand}[0,1) \rightarrow z_i = 1$

else $z_i = -1$

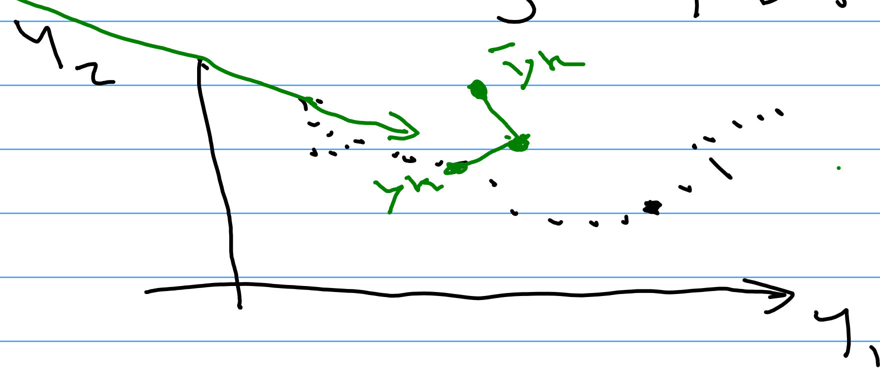
$$E(y\,z) = -z^T W y$$



clamped phase y^k z^k

unclamped $\bar{y}^k$ $\bar{z}^k$

$$w_{ij} \leftarrow w_{ij} + \eta\left[y_j^k z_i^k - \bar{y}_j^k \bar{z}_i^k\right]$$



L2 regularization on weights

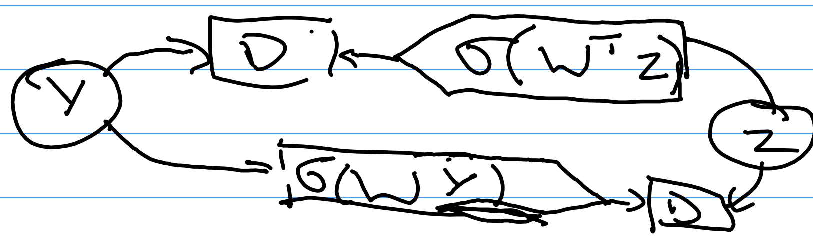
CD-1

Persistent CD

$$E(Y, Z) = -Z^T W Y$$

$$\frac{1}{2}\left[D(Z, WY) + D(Y, W^T Z)\right]$$

$$D(A, B) = -A^T B$$



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